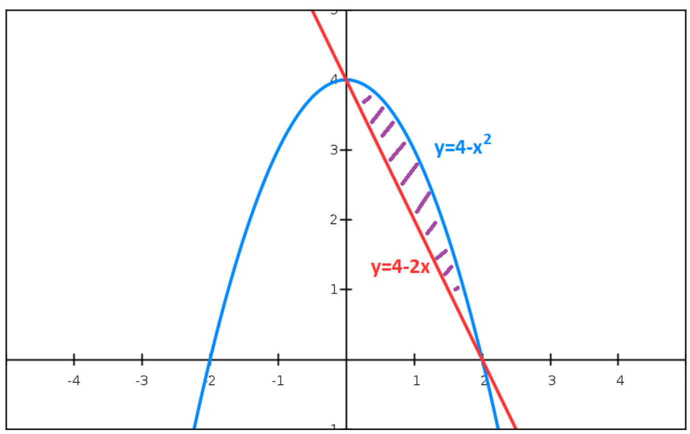
QUIZ 9 SOLUTIONS: LESSONS 11-12
SEPTEMBER 25, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] Find the area bounded by the curves $y = 4 - x^2$ and $y = 4 - 2x$.

   Solution: Our graph looks like



   We are not given the bounds, so we need to find them. Write

$$4 - x^2 = 4 - 2x$$
$$\Rightarrow \quad 0 = x^2 - 2x$$
$$0 = x(x-2)$$

The solutions of this are $x = 0$ and $x = 2$. Thus, our bounds are $0 \le x \le 2$. If we test at $x = 1$, we see that

$$4 - (1)^2 = 4 - 1 = 3$$

and

$$4 - 2(1) = 4 - 2 = 2$$

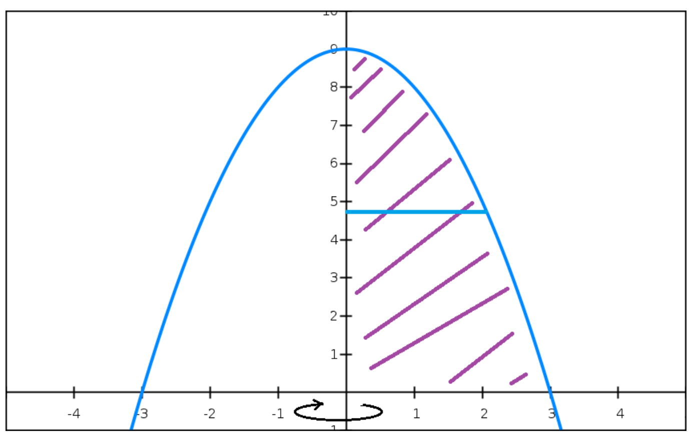
which means that $y = 4 - x^2$ is the larger function on the interval $0 \le x \le 2$. Our area is then given by

$$\begin{aligned}\text{Area} &= \int_0^2 \big(4 - x^2 - (4 - 2x)\big)\, dx \\ &= \int_0^2 (2x - x^2)\, dx\end{aligned}$$

$$= \frac{2}{2}x^2 - \frac{1}{3}x^3\Big|_0^2$$
$$= (2)^2 - \frac{1}{3}(2)^3$$
$$= 4 - \frac{8}{3}$$
$$= \frac{12}{3} - \frac{8}{3}$$
$$= \boxed{\frac{4}{3}}$$

2. [6 pts] Find the volume obtained by revolving the region given by the function $y = 9 - x^2$ in the first quadrant around the y-axis.

Solution: Our graph looks like



We are given $y = 9 - x^2$, but since we are revolving around the y-axis, we need to solve for x. Write

$$y = 9 - x^2$$
$$\Rightarrow \quad x^2 + y = 9$$
$$\Rightarrow \quad x^2 = 9 - y$$
$$\Rightarrow \quad x = \sqrt{9-y}$$

where we take the positive root because we are to the right of the y-axis. We also see that our bounds are $0 \leq y \leq 9$. Therefore, our volume is given by

$$\text{Volume} = \int_0^9 \pi\left(\sqrt{9-y}\right)^2\, dy$$
$$= \int_0^9 \pi(9-y)\, dy$$

$$= \pi \left[9y - \frac{1}{2}y^2\right]_0^9$$

$$= \pi \left[9(9) - \frac{1}{2}(9)^2\right]$$

$$= \pi \left[81 - \frac{81}{2}\right]$$

$$= \boxed{\frac{81\pi}{2}}$$